# Dynamic Adverse Selection and Asset Sales*

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**Abstract**

This paper presents a dynamic adverse selection model in the OTC market with bilateral trading. Investors meet over-the-counter to trade heterogeneous assets under asymmetric information. The cream-skimming effect emerges due to the heterogeneous sophistication among buyers, where the low-type seller strategically forgoes trading opportunities with gains from trade in order to take advantage of the unsophisticated investors in the market. When the market is pessimistic, time to sale increases in asset quality, heterogeneous sophistication improves market liquidity; when the market is optimistic, time to sale decreases in asset quality, cream-skimming incentive endogenously occurs, which reduces the trading efficiency. The implications and predictions on initial public offerings, real estate market are discussed in the paper.

**JEL classification**: C73, C78, D82, G12, G14
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# 1 Introduction

In financial markets with adverse selection, time to sale conveys valuable information about the asset quality. In the mortgage market, investors in mortgage-backed securities cannot

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observe the quality of underlying portfolios of mortgages, but they can update the quality through the time it takes between origination and securitization. In the IPO market, potential investors can infer the firm's prospectus through the time it takes to go IPO. In the real estate market, potential buyers can infer the house quality through how long the house has been listed on the market. In the labor market, the employer can not observe the ability of the employee, but he can infer the employee's ability through the unemployment duration.

On the theoretical side, dynamic signaling models are widely applied to understand the role of time to sale. Many recent studies (Taylor, 1999; Kremer and Skrzypacz, 2007; Fuchs and Skrzypacz, 2013) view time to sale as a signaling device. The main logic is as follows: in a market with privately informed sellers, high type sellers, who have lower waiting cost or higher outside option of no trade, have an incentive to delay trade to signal the quality. Therefore, time to sale can serve as a signal of quality, i.e., the longer it takes for the asset to be traded, the more likely that the asset is of high quality.

On the empirical side, however, the evidence about the time to sale is mixed. In the mortgage market, Adelino et al. (2019) and Klee and Shin (2020) find that mortgage quality improves [1] as the length of time increases between origination and securitization[2]. In the IPO market, Fuchs et al. (2016) and Gratton et al. (2018) find that high-quality firms wait longer before IPO[3]. In the labor markets, negative duration dependence is widely documented; that is, a longer unemployment spell is associated with a lower job finding rate and lower wages(Clark et al., 1979; Kroft et al., 2013; Jarosch and Pilossoph, 2019); in the real estate market, time-on-the-market is negatively correlated with the sale prices. (Krainer, 2001; Krainer et al., 2008; Dubé and Legros, 2016).This suggests that time to sale is negatively correlated with quality in these markets. The distinct predictions of time to sale in different markets are puzzles and cannot be explained by the extant models within the same framework.

The heterogeneous sophistication among buyers is nontrivial in asset sales and can play a key role in understanding the trading in the OTC market. Many assets, such as corporate bonds and derivatives, are traded over-the-counter (OTC). Sellers of assets are heterogeneous and have private information about the quality of their assets as well as their own valuation. Buyers of assets are heterogeneously sophisticated too. Some of these buyers, such as venture capitalists, hedge funds, trusts, and investment banks, are more sophisticated so that they may know more than other buyers about the quality of assets for sale. [4]. In the IPO

[1]They use the default rate as the measure of mortgage performance

[2]This is also called the skimming property in the literature, which states that time to sale of an asset increases in quality

[3]Fuchs et al. (2016) use the post-IPO profitability as a measure of quality

[4]The better sophistication may be due to better information acquisition technology or larger information

market, institutional investors are better informed about the firm's prospectus and gain significant profits from IPO(Chemmanur et al., 2010). In the real estate market, some investors are better sophisticated than others due to their expertise or familiar with the neighborhood(Stroebel, 2016; Levitt and Syverson, 2008). Can the heterogeneity in investor sophistication explain the puzzle of time to sale in different markets? How does the share of sophisticated investors shape the trading dynamics?

To explain the empirical puzzle on the relation between time to sale and asset quality, I introduce investors' heterogeneous sophistication into a dynamic adverse selection model in the OTC market with bilateral trading. The key result of my model is that a cream-skimming effect can emerge when the investors are heterogeneously sophisticated. When the perceived asset quality is high on the market, unsophisticated buyers are willing to pay a pooling price for the average asset. Then matching with an unsophisticated investors is more profitable since they can get a better offer. Therefore, the low-type seller has incentives to gamble on meeting with unsophisticated investors by rejecting trading opportunities with sophisticated investors. Thus, trading between the low-type seller and sophisticated investors completely break down. This "gambling" incentive for the low-type seller can generate a negative relationship between time to sale and asset quality. These results provide a new angle to understand the negative duration dependence phenomenon in the labor market and the real estate market.

Now I will introduce the key ingredients of the model. A seller (she) has one unit of the indivisible asset to sell due to liquidity reason, and the asset quality could be high or low. Two groups of buyers are characterized by their sophistication level. Sophisticated investors possess better information about asset quality than unsophisticated investors. In a bilateral OTC meeting, a seller searches buyers sequentially, and once met, the buyer (he) makes a take-it-or-leave-it offer to the seller. The offers are private in the sense that offers history is not observable to subsequent buyers[5]. The buyer faces adverse selection because he is uninformed about the asset quality if he is unsophisticated. He makes either a pooling offer, a separating offer, or randomizes between both. The pooling offer targets sellers who own a high-quality asset. Given any price accepted by the high-type seller, under asymmetric information, owners of low-quality assets could also trade at this price and receive information rents. To avoid paying these rents, the unsophisticated buyer can instead bid a separating offers for low-quality assets, but he would forgo the gains from trade with the high-type seller. The bidding strategy for unsophisticated buyers depends on the perceived asset quality on

process capacity (Kacperczyk et al., 2019)

[5]Private offer assumption is to capture the opacity in OTC market, it also captures the widely adopted non-disclosure agreement(NDA) in financial markets

the market, the more optimistic about the asset quality, the more likely he will bid a pooling offer.

In the model, the correlation of time to sale and asset quality relies on the market belief about the asset quality. When the market is pessimistic, i.e., the market belief is low, then time to sale is positively correlated with asset quality due to the signaling effect; when the market is optimistic, i.e., the market belief is high, then time to sale is negatively correlated with asset quality due to cream-skimming effect. Signaling effect together with the cream-skimming effect can explain the empirical puzzle on time to sale.

The first dynamic force of the model is a *signaling effect* whereby high-type seller signals her quality by turning down offer from unsophisticated investors. When the perceived asset quality is low on the market, unsophisticated buyers are reluctant to pay a pooling price for the asset, therefore, high-type seller only trades with sophisticated buyers with high valuation. On the other hand, the low-type seller's waiting value depends on whether the unsophisticated investors are willing to pay a pooling offer. When the unsophisticated buyers are pessimistic, they are unwilling to pay a pooling price, therefore, waiting is too costly for the low-type and she prefers to trade earlier. The trading rates difference between high-type and low-type seller provides a signaling device for the high-type seller, the longer the asset stays on the market without trading, the more likely the asset is high quality. This signaling effect can generate positive relationship between time to sale and asset quality.

The second dynamic force in the model is a *cream-skimming* effect whereby low-type seller is only willing to trade with unsophisticated buyers and forgo trading opportunities with sophisticated buyers. When the perceived asset quality is high, the unsophisticated investors are willing to trade at pooling price. Anticipating the potential pooling offers from the unsophisticated investors under favorable market belief, the low-type seller is reluctant to trade with the sophisticated buyers at separating price as her waiting value is higher than buyers' valuation for low-quality asset. Thus, the trade between the low-type seller and the sophisticated buyers breaks down, and the low-type seller cream-skims the unsophisticated buyers on the market. This cream-skimming effect generates positive correlation between the asset quality and time to sale, in particular, it is the low-type seller who is more likely to wait in order to gamble on matching with unsophisticated investors. I also identify the condition when the cream-skimming effect can exist (Assumption 2). Cream-skimming effect requires that gambling on matching with unsophisticated investors for the low-type seller is not too costly. Thus, in the market with less search friction and less sophisticated investors, the cream-skimming effect is more likely to exist. The search friction and ratio of sophisticated investors difference in different market can explain the empirical puzzle on the time-to-sale.

Heterogeneous sophistication among buyers shape the trading dynamics in the following ways. First, it improves trading efficiency when the market is pessimistic. Sophisticated investors are better informed, and they can identify good assets in the market, their participation helps boost trading volume and improve trading efficiency. Unlike the static lemon model (Akerlof, 1970) where the market breaks down completely, in the presence of sophisticated investors, the information asymmetry between sophisticated buyers and sellers is not severe enough such that trading between sophisticated buyers and the high-type seller is still active. This mechanism is reminiscent of the mechanism in Glode and Opp (2016)which information intermediary can reduce the asymmetric information between sellers and buyers and improve trading efficiency. Second, it provide incentives for the low-type seller to cream-skim the unsophisticated investors. The cream-skimming effect relies on the heterogeneous sophistication among buyers, where buyers with heterogeneous sophistication have different valuation for the same asset. When the valuation difference is large enough, then low-type seller is willing to take advantage of this valuation difference by cream-skimming only the unsophisticated investors. And this effect disappears when the buyers are homogeneously sophisticated.

In the model, trading volume, defined as the average trading rate, deteriorates over time regardless of the perceived asset quality. And this deterioration of trading volume is due to the learning effect from time to sale. The trading volume depends on the relative trading rates for both types of asset and market belief about asset quality. The more aggressively the high-quality assets are traded in the market, the less likely the asset remained on the market is high-quality, therefore the average trading rate for the asset remained on the market decreases over time.

In the extension, I consider the quality shock[6] to the asset where the asset quality deteriorates over time. And I find that when the quality shock is not severe, the main mechanism in the paper is still robust, and the net effect of time to sale on asset quality depends on the market belief about the asset quality. I can still get positive correlation between time to sale and asset quality when the market is pessimistic and negative correlation when the market is optimistic. However, when the quality shock is severe, then the prediction of time to sale and asset quality is purely driven by the quality shock, and only negative correlation between time to sale and asset quality can exist.

[6] In real estate market and labor market, the negative correlation between time to sale and asset quality could be due to the deterioration of asset quality over time.

## 1.1 Related Literature

This paper builds on the large literature on adverse selection initiated by the seminal work of Akerlof (1970). Among many other papers, Swinkels (1999), Janssen and Roy (2002) Kremer and Skrzypacz (2007),Daley and Green (2012),Camargo and Lester (2014) analyze the dynamic versions of the lemon market models in centralized market or decentralized market where sellers are better informed than buyers. Janssen and Roy (2002) show that when sellers are endowed with fixed private information, equilibrium prices increase over time as owners of higher type assets delay trade in order to signal their type. While all assets are eventually sold, trade is delayed, and the market equilibrium is therefore inefficient. Camargo and Lester (2014) find related market dynamics in a decentralized market where buyers and sellers are matched randomly each period. As in the centralized market, Janssen and Roy (2002), trade is delayed, and prices and averages quality increase over time. introduce exogenous information into a dynamic signaling model with private offers. In their model, a grade is revealed at some fixed time, provided that trade has not already occurred. In contrast to Swinkels (1999), trade is always delayed with positive probability. A critical insight for their work is that noisy information causes an endogenous market for lemon to develop. In equilibrium, trade breaks down completely just before revelation of the information. Daley and Green (2012, 2016) consider a model in which public news affect quality of a seller's asset is released over time and construct an equilibrium that involves periods of complete market break down. My paper contributes to theoretical literature on dynamic adverse selection. In particular, I follow the line of Swinkels (1999), Zhu (2012) and Daley and Green (2012, 2016) by assuming that investors do not observe previous offers from earlier buyers. Unlike Swinkels (1999) who considers a model in which the lemon condition is not binding, and Daley and Green (2012, 2016) who consider exogenous information revelation, I primarily investigate how does the investor sophistication affects the trading efficiency and asset prices.

This paper is also related to work by Daley and Green (2012, 2016), who study the role of exogenous "news" on dynamic adverse selection models. In my paper, the "news" is endogenous, and it mainly comes from the sellers' trading strategies.

My paper is also related to the literature of competitive search models with asymmetric information in the OTC market (Guerrieri et al., 2010; Guerrieri and Shimer, 2014; Chang, 2018). In this literature, the trading rate is a signaling device, where high-quality asset prefers trading in a less liquid market in order to separate from the low-quality asset. The trading rate in my model is a learning device, and different types of asset cannot be traded in different sub-markets, thus, high-type sellers cannot fully separate themselves through trading rate.

My model contributes to the literature of cream-skimming in the financial market. Fishman and Parker (2015), Bolton et al. (2016), Romanyuk and Smolin (2019),Zou (2019) and Vallee and Zeng (2019) show that sophisticated investors can obtain a large share of rent by cherry-picking good assets due to their information advantage. My paper differs with the cream-skimming literature in the following aspects. First, in my model, it is the low-type seller who chooses to cream-skim the unsophisticated buyers in the market, while in the literature, the informed buyers cream-skim the less informed buyer by cherry-picking the good asset in the market. Second, the the quality of assets remaining in the market could deteriorate or ameliorate over time depending on the market belief, while in the cream-skimming literature, they predict that the asset quality deteriorates over time.

My paper is also related to other strands of literature with developing adverse selection(Martel et al., 2018; Hwang, 2018). This literature assumes that the initially there is no asymmetric information between sellers and buyers, and sellers learn from holding the asset. The endogenously developed asymmetric information can generate U-shape equilibrium where the trading price and probability drop at the beginning and then increase over time. My paper is different with literature

The most related paper with my mine is Frenkel (2020), who considers dynamic asset sales in the OTC market with a feedback effect. In his model, the seller cares about the gains from trade through the transaction, and the information revealed during the asset sales. And he finds that the market views event of no sales as a bad signal about the asset quality, which is consistent with my model under the high market belief, but the mechanism in these two papers is different. In his model, the low-type seller is reluctant to immediately sell the low-quality asset due to the negative information revealed through asset sales. In my model, the seller's outside option depends on the market belief and relative size of sophisticated investors. When the market is optimistic, the low-type seller values more about the asset than buyers' willingness to pay. Thus, low-type seller has an gambling incentive to match with an unsophisticated investors.

This paper is also related to asset sales in the financial market. Edmans and Mann (2019) consider the firm's financing through asset sales. They distinguish between core-asset and non core-asset, and claim that by non core-asset is not information sensitive, it has less impact on the stock price due to less asymmetric information. Bond and Leitner (2015) considers the asset sales with multiple units. They show that due to the existence of multiple units of assets, sellers in the market care about the price of the asset sold and the value of their inventories. Tefrenkel2020, they find that firms may choose not to trade to avoid revealing bad news about the value of the inventory. This paper provides another explanation of trade delay when the seller cares about the market price he can get and the value of their inventory. In my model, only one single indivisible asset is for sale, and there is no effect on the inventory. The trade delay occurs mainly due to two reasons. When the initial quality of the asset pools is low, unsophisticated buyers are pessimistic about the asset quality, and they are not willing to pay a pooling offer, thus, the seller with the high-type asset does not trade due to his outside option; when the initial quality of asset pools is high, unsophisticated buyers are optimistic about the asset quality, and they are willing to bid pooling price. Anticipating this, the low-type seller is reluctant to trade with sophisticated buyers since the low-type seller's waiting value is higher than the sophisticated buyer's valuation of the low-quality assets. Thus, the trade delay is mainly due to the strategic delay from the low-type seller.

The paper is organized as follows: in Section 2, I describe the environment. In Section 3, I present preliminary analysis. In Section 4, I characterize the unique stationary equilibrium. In Section 5, I characterize the fully dynamic non-stationary equilibrium. In Section 6, I consider some comparative statics in the model. In Section 8, I present some empirical implications of the model. Section 9 concludes.

## 2 The Model

**Environment.** I consider a dynamic trading game between a long-lived seller (she) and a sequence of short-lived buyers (he), all agents are risk-neutral and discount future cash flow at same rate $r > 0$. Time, denoted by $t$, is continuous and runs from zero up to infinity $t \in [0, \infty)$. There is one unit of indivisible asset with quality $\theta \in \{H, L\}$. I refer to a type-H asset as "high-quality" and to a type-L as "low-quality". The common prior probability that the asset is high quality is $\pi_0 \in (0, 1)$. At $t = 0$, the seller is endowed with the single indivisible asset and the asset type is her private information.

Type-$\theta$ asset delivers cash flow $c_\theta(v_\theta)$ to the seller (buyer) if she (he) owns the asset. Define the holding value of type-$\theta$ asset as $V_\theta = v_\theta/r$ to the buyer and $C_\theta = c_\theta/r$ to the seller. $C_\theta$ can be interpreted as the outside option for type-$\theta$ seller by holding the asset forever. Throughout this paper, I assume that

**Assumption 1.** $v_H > c_H > v_L > c_L$

Assumption 1 says that the the assets are more productive if held by buyers rather than sellers ($v_\theta > c_\theta$), and gains from trade are positive for both type of assets. The standard lemon condition[7] holds in the model ($c_H > v_L$), that is, it is not always profitable for the buyer to make a pooling offer to the buyers.

**Buyers**. A continuum of buyers measures one in the market of two types $z \in \{S, U\}$ , where $S$ stands for sophisticated buyer and $U$ stands for unsophisticated buyer. With fraction $s \in (0, 1)$ of buyers are sophisticated, and they are perfectly informed about asset quality owned by the seller[8], with fraction $1 - s$ of the buyers are unsophisticated, they only observe how long the asset has been traded on the market. Buyers' types are their private information and not observable to the seller.

**Trading protocol**. The market is over-the-counter (OTC). In a bilateral OTC meeting, the seller searches buyers sequentially with no cost[9]. Buyers arrive at random times that correspond to the jump times of a Poisson process with intensity $\lambda$. Once meet with seller, the buyer makes a take-it-or-leave-it (TIOLI)[10] private offer to the seller and then the seller decides whether to accept it or not. The offers are private in the sense that the search history and previous rejected offers are not observable to subsequent buyers. If the seller accepts the offer, then the game ends. Otherwise, the buyer leaves the market the seller continues to search for a new buyer[11].

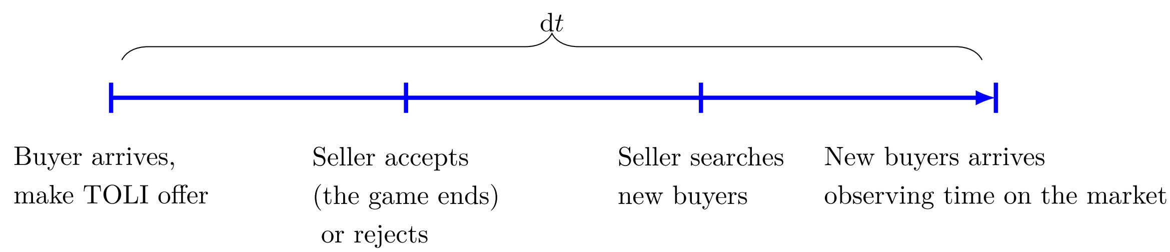



Figure 1: Sequence of Events

**Discussion of Assumptions**: The model entails a number of simplifying assumptions, which are made primarily to facilitate a tractable analysis and to keep the intuition for the

[7]This assumption (see also in Daley and Green (2012) is stronger than the lemon condition in static adverse selection model where the lemon condition is $\pi_0 V_H + (1 - \pi_0)V_L < C_H$, and $\pi_0$ is the initial market belief

[8]I model the sophisticated investors as the ones with better information. In the behavior finance literature, some papers model sophisticated investors as rational investors, and unsophisticated investors as irrational investors, see (Ljungqvist et al. (2006))

[9]This zero-cost assumption allows me to bypass the Diamond paradox (Diamond (1971)) and focus on the sequential nature of search, rather than the pecuniary cost of search.

[10]In the extension, I consider competitive buyers in the market, and the result is robust to competitive buyers in the market.

[11]Unlike Zhu (2012), repeat contacts in my model have zero probability since there is a continuum of buyers with measure 1. No repeated contacts is a key feature in search model with infinite number of buyers (Duffie et al., 2005, 2007; Vayanos and Wang, 2007; Vayanos and Weill, 2008)

main forces accessible. One of the key assumption of the model is that the trading history is private (opaque). The assumption here is that in each time $t$ where no sale takes place, the market cannot tell whether this is because exogenous reasons (no match due to search friction) or because of the seller's decision not to sell. If however, that the seller's decision not to sell always become public, then in equilibrium, high-quality asset owners can signal their asset quality by turning down unfavorable offers. The private offer assumption in the model is meant to capture in a simple way trading opacity in OTC markets where assets transit through dealers before reaching end customers. Another interpretation of the private offer assumption is the widely adoption of nondisclosure agreements(NDAs) in real estate market and Venture Capital market. The NDAs are used to protect the confidential information about the seller (home owner, startups ). Private offer is assumption is only relevant when the gains from trade is positive for low-quality asset. With zero gains from trade, then low-type asset can only be traded together with high-quality asset, thus, trading history does reveal any information.

Another key assumption in the model is that the seller cannot commit to when to sell the asset, which is common in the literature of dynamic adverse selection. Unlike the the static signaling models in financial market (Leland and Pyle, 1977), sellers cannot signal through the timing of the asset sales (or no sales ) due to the search friction. Given any time $t$, the probability that seller has not met with the buyer is strict positive, therefore, any commitment of sale before $t$ is not feasible. Commitment issue is the main difference between the static model and dynamic model which is first discussed by Admati and Perry (1987) in the context of education signaling.

## 2.1 Strategies

In this subsection, I will discuss the strategies and equilibrium definition. First, I will introduce some notations to describe the buyers' strategy in the full dynamic game, allowing for both pure and mixed strategy.

The strategy for the buyer is a mapping from his information set to a distribution of price offers. I allow mixed strategies for both sellers and buyers in the model. And the strategy for the seller is a mapping from her type and the price offer received to the probability of acceptance. The (Unsophisticated) buyers' belief is represented by a function $\pi : \mathbb{R}_+ \rightarrow \Delta\{H, L\}$, which maps the calendar time to the probability that the asset is good quality. For the sophisticated investors, they can observe $\theta$ perfectly, therefore, the offer strategies of type-$S$ buyers in a matching with type-$\theta$ seller are represented by a mapping $\sigma_t^{S,\theta}$ from $\mathbb{R}_+$ to a set of probability distribution over $\mathbb{R}$, where $\sigma_t^{S,\theta}(p)$ denotes the probability that the type-S buyer's offer in a match at time t with type $\theta$ seller is lower than $p$. The offer strategies of type-$U$ buyers are represented by a mapping $\sigma_t^U$ from $\mathbb{R}_+$ to a set of probability distribution over $\mathbb{R}$, where $\sigma_t^U(p)$ denotes the probability of type-$U$ buyer's offer in a match at time $t$ is lower than $p$. The acceptance strategy of for type-$\theta$ seller at time $t$ is represented by a function $\mu_t^\theta : \mathbb{R}_+ \to [0,1]$ where $\mu_t^\theta(p)$ is the type-$\theta$ seller's acceptance decision at time $t$. Let $\mu_t^\theta(p) = 0$ denote the type-$\theta$ seller's decision to reject the offer $p$ and $\mu_t^\theta(p) = 1$ denote the seller's decision to accept the offer $p$.

Given the strategies for buyers and sellers, denote $\lambda_t^\theta$ as the probability that a type-$\theta$ seller trades at time $t$.

$$\lambda_t^\theta = \lambda \left( s \underbrace{\int_p \mu_t^\theta(p)\, \mathrm{d}\sigma_t^{S,\theta}(p)}_{\text{prob trading with S}} + (1-s) \underbrace{\int_p \mu_t^\theta(p)\, \mathrm{d}\sigma_t^{U}(p)}_{\text{prob trading with U}} \right) \tag{1}$$

Trading rate for each type of asset captures the likelihood that type-$\theta$ get traded in the short interval between $[t, t+\mathrm{d}t]$ conditional no sales before $t$. There two components in the trading rate for type-$\theta$ asset. The first term captures the probability trading with sophisticated buyers, since sophisticated buyers are perfectly informed about the asset quality, their offers are type-dependant. The second term captures the probability trading with unsophisticated buyers. One can interpret the trading rate $\lambda_t^\theta$ as the market liquidity (or trading volume) for type-$\theta$ asset [12], higher trading rate means type-$\theta$ asset class is more liquid and vice verse.

In a dynamic trading game with search friction, we say the trade is *efficient* if any pair of buyer and seller with positive gains from trade trades with each other with probability one. In our game, the trading is fully efficient if and only if

$$\lambda_t^H = \lambda_t^L = \lambda \tag{2}$$

## 2.2 Belief

For the sophisticated investors, they can observe $\theta$ perfectly. For the unsophisticated investors, they only observe how long the asset has been traded.

Therefore, given the strategies $\{\sigma_t^{S,\theta}\}_{\theta\in\{H,L\}}, \sigma_t^U, \mu_t^\theta$ their perceived quality of asset follows

[12]In this model there is only one unit of asset, and the trading game stops once the asset is traded, thus, we can also interpret the $\lambda_t^\theta$ as the trading volume for type–$\theta$ asset at time $t$.

$$\pi_t = \frac{\pi_0 e^{-\int_0^t \lambda_s^H \,\mathrm{d}s}}{\pi_0 e^{-\int_0^t \lambda_s^H \,\mathrm{d}s} + (1-\pi_0) e^{-\int_0^t \lambda_s^L \,\mathrm{d}s}} \tag{3}$$

where $\lambda_t^\theta$ is the trading rate for type-$\theta$ asset shown in (1). The numerator for (3) is the probability that the high-type asset has not been traded before $t$, while the denominator is the probability that there is no sales before $t$.

Rewrite (3) recursively, and take $\mathrm{d}t \to 0$, I can obtain the following differential equation

$$\mathrm{d}\pi_t = \pi_t(1-\pi_t)(\lambda_t^L - \lambda_t^H)\,\mathrm{d}t \tag{4}$$

with boundary condition $\pi_0$ given.

The dynamics of the belief process is intuitive. As long as the asset has not been traded, the belief about asset quality depends on the relative trading rate of different types of assets. If buyers believe that the high-type asset is traded more aggressively on the market, then the asset remains on the market is more likely to be low-quality. On the contrary, if the low-type seller asset is trading more aggressively, then conditional on no sales, the market will revise their belief upwards. The learning speed $(\lambda_t^L - \lambda_t^H))$ is proportional to the search intensity, the higher the search intensity, the faster information is revealed to the market.

## 2.3 Continuation Value

If a seller of type-$\theta$ rejects the offer from buyer at time t and plans to accept an offer at (random) time $\tau > t$, then her expected payoff is $\mathbb{E}\left\{e^{-r\tau}\left(\tilde{p}_\tau - C_\theta\right\}) + C_\theta\right\}$ , where $\tau$ is the trading time and $p_\tau$ is the trading price. Given the strategies taken by the seller and buyers, it can generate the distribution of trading time and trading price, and then I can define the continuation value to type-$\theta$ seller if she decides to wait at time $t$.

$$W_t^\theta = \mathbb{E}\left\{\left(1-e^{-r(\tau-t)}\right)C_\theta + e^{-r(\tau-t)}\tilde{p}_\tau\right\} \tag{5}$$

There are two components in the continuation value. Before trading at time $\tau$, type-$\theta$ seller receives the cash flow by holding the asset. $(1-e^{-r(\tau-t})C_\theta$ captures all the cash flows received between $[t,\tau]$. The second term is the payoff from trading the asset in $\tau$ at price $p_\tau$. The expectation is taken on all the potential trading time and trading prices generated by the strategies taken by the agents.

For the high-type seller, the continuation value is straightforward. On the one hand, the continuation value of sellers with high-quality asset is at least $C_H$ since the sellers always have the option to hold on to their assets, and $C_H$ is the outside option for high-type seller. On the other hand, no buyer will offer a price higher than $C_H$ in equilibrium since the buyer has all the bargaining power. Therefore,

$$W_t^H = C_H \tag{6}$$

Therefore, I can focus exclusively on the continuation value for the low-type seller to characterize the equilibrium.

## 2.4 Equilibrium Definition

**Definition 1.** *A Perfect Bayesian Equilibrium is a quadruple* $\left\{\sigma^S, \sigma^U, \mu^\theta, \pi\right\}$ *such that*

1. *Given* $\mu^\theta, \theta \in \{H, L\}$ *type-S buyer chooses* $\sigma_t^{S,\theta}$ *to maximize*

$$\int (V_\theta - p)\mu_t^\theta(p)\, \mathrm{d}\sigma_t^{S,\theta}(p) \tag{7}$$

2. *Given* $\mu^\theta, \theta \in \{H, L\}$ *type-U buyer chooses* $\sigma_t^U$ *to maximize*

$$\mathbb{E}\left\{\int (V_\theta - p)\mu_t^\theta(p)\, \mathrm{d}\sigma_t^U(p) \mid \pi_t\right\} \tag{8}$$

3. *Given* $\sigma_t^z, z \in \{S, U\}$ *, type-*$\theta$ *seller chooses* $\mu_t^\theta$ *to maximize*

$$W_t^\theta + s\int_p (p - W_t^\theta)\mu_t^\theta(p)\, \mathrm{d}\sigma_t^{S,\theta}(p) + (1-s)\int_p (p - W_t^\theta)\mu_t^\theta(p)\, \mathrm{d}\sigma_t^U(p) \tag{9}$$

4. *Belief Consistency: Conditional on no trading, the belief follows Bayes' rule* (**??**)

# 3 Equilibrium Analysis

In this section, I will provide some preliminary analysis. First, it is easy to see that the high-quality seller always trades with informed buyers when they meet due to the gains from trade. The existence of informed buyers in the model is the key of providing information without any transaction. In the next lemma, I will show that the continuation value for the high-quality seller is always her valuation for the asset.

**Lemma 1.** *In equilibrium, the optimal acceptance decision for each type of seller $\theta \in \{H, L\}$ is given by*

$$\mu_t^\theta(p_t) = \begin{cases} 0 & \text{if } p_t < W_t^\theta \\ [0,1] & \text{if } p_t = W_t^\theta \\ 1. & \text{if } p_t > W_t^\theta \end{cases} \tag{10}$$

*Proof.* All proofs are omitted and shown in Appendix A □

Lemma 1 states that the optimal acceptance strategy for each seller is a threshold strategy, it depends on the price offered and the continuation value for the seller. Note that $W_t^\theta$ is continuous in t because the probability that the buyer arrives at a given time interval vanishes as the length of the time interval shrinks to zero.

Given the cut-off strategy taken by the seller, the payoff to the unsophisticated buyer if he bids $p$ is

$$\pi_t(V_H - p)\mathbb{I}(p \geq C_H) + (1 - \pi_t)(V_L - p)\mathbb{I}(p \geq W_t^L) \tag{11}$$

**Lemma 2.** *In equilibrium, three types of prices $p_t^l = W_t^L$, $p_t^h = C_H$ and $p_t^n \geq V_L$ are offered from the unsophisticated buyers.*

In the remaining paper, we call $p_t^l$ as the separating offer, which is only accepted with positive probability by low-type seller, and $p_t^h$ as the pooling offer, which is accepted by sellers with probability 1, and $p_t^n$ as non-serious offer, which is rejected for sure by both types of sellers.

The intuition is straightforward: in order to target the high-type seller, the optimal (lowest) offer made is the $C_H$, similarly in order to target the low-quality asset, the optimal (lowest) offer made is the reservation value for the low-type. However, the continuation value for the low-type seller could be higher than buyer's valuation for the low-quality asset, therefore, non-serious offer $p_t^n$ is made by the unsophisticated buyers.

Given above lemma, we can characterize the continuation value dynamics for low-type seller. Denote $\gamma_t \equiv (1 - s)\sigma_t^U(C_H)$ as the probability that the low-type seller receives offer $C_H$ [13]. Then the continuation value dynamics follows

$$W_t^L = rC_L\,\mathrm{d}t + e^{-r\,\mathrm{d}t}\left\{\gamma_t\,\mathrm{d}tC_H + (1 - \gamma_t\,\mathrm{d}t)W_{t+\mathrm{d}t}^L\right\} \tag{12}$$

[13] In Maurin (2020), $\gamma_t$ is interpreted as the measure of market liquidity

We can rewrite (12) as the differential equation such that

$$rW_t^L\,\mathrm{d}t = \underbrace{rC_L\,\mathrm{d}t}_{\text{cash flow}} + \underbrace{\gamma_t(C_H - W_t^L)\,\mathrm{d}t}_{\text{trading opportunity}} + \underbrace{\mathrm{d}W_t^L}_{\text{capital gains}} \tag{13}$$

The LHS of (13) is the required return for holding the asset, and first term on the RHS of (13) is the cash flow by holding the asset between $[t, t+\mathrm{d}t]$, the second term captures the possibility to get $p^h = C_H$ from buyers, where $\gamma_t$ is the probability of seller with low-quality asset to receive a pooling offer ($C_H$) from unsophisticated investors.

## 3.1 Optimal bidding for buyers

In this section, we will discuss the optimal bidding for the buyers. For the sophisticated buyers, they can observe $\theta$, therefore the bidding strategy is given by

$$p_t^S = \begin{cases} C_H & \text{if } \theta = H \\ \min\{W_t^L, V_L\} & \text{if } \theta = L \end{cases} \tag{14}$$

Basically, the informed buyers bid $C_H$ for high-quality asset, and $W_t^L$ for low-quality asset when it is profitable, otherwise, he will bid a non-serious offer. Next, we focus on the optimal bidding for the low-quality seller.

If the unsophisticated investor is targeting both types of sellers in the market, then his payoff is

$$\pi_t(V_H - C_H) + (1-\pi_t)(V_L - C_H) \tag{15}$$

if he targets only the low-type seller in the market, then his payoff is

$$(1-\pi_t)(V_L - W_t^L) \tag{16}$$

and if he makes a non-serious offer, then the payoff is 0.

**Lemma 3.** *The optimal offering strategy for the uninformed buyer is characterized by a threshold belief*

$$\bar{\pi}_t = \frac{C_H - \min\{V_L, W_t^L\}}{V_H - \min\{V_L, W_t^L\}} \tag{17}$$

*such that*

*(i) If $\pi_t > \bar{\pi}_t$, then unsophisticated buyers make a pooling offer $C_H$.*

*(ii) If $\pi_t < \bar{\pi}_t$ and $V_L > W_t^L$, then unsophisticated buyers make a separating offer $W_t^L$*

*(iii) if $\pi_t < \bar{\pi}_t$ and $V_L < W_t^L$, then unsophisticated buyers make a non-serious offer $p < W_t^L$.*

In Lemma 3, $\bar{\pi}_t$ is the cut-off belief such that pooling offer is optimal and guarantees a non-negative payoff to the buyer. The intuition for Lemma 3 is as follows: when the market belief about the asset quality is high enough, make pooling offer ($C_H$) to target both types of seller is optimal and profitable. When the market belief is below the threshold belief, and low-type seller values more by holding the asset, then separating offer is optimal and profitable. However, when market belief is below the threshold belief, and low-type seller's continuation value is above the buyer's valuation for low quality asset, then no offer can be accepted with positive probability yet still provides non-negative profit to the buyer. Thus, in this case, only non-serious offers are made by the unsophisticated buyers.

For the remaining paper, I assume that parameters in the paper satisfy

**Assumption 2.** *Assume*

$$\frac{rC_L + \lambda(1-s)C_H}{r + \lambda(1-s)} > V_L \tag{18}$$

One way to understand Assumption 2 is to consider a naive strategy from the unsophisticated buyers. In the naive strategy, the unsophisticated buyers always bid $C_H$ regardless of his belief about the asset quality. Given the naive strategy, the continuation value to the low-type seller if only targets unsophisticated buyers is

$$\bar{W}^L = \frac{rC_L + \lambda(1-s)C_H}{r + \lambda(1-s)} \tag{19}$$

Assumption 2 states that, given the naive strategy taken by the unsophisticated investors, the low-type seller has incentive to gamble on meeting with unsophisticated buyers in the market. In the following lemma, we will show that under Assumption 2, fully efficient trading is not feasible.

**Lemma 4.** *If Assumption 2 holds, equilibrium with efficient trading is not feasible.*

Lemma 4 states that in any equilibrium under Assumption 2, the trading in the market cannot be efficient. The intuition is as follows: in order to achieve efficient trading, a pooling offer is required from the unsophisticated buyers. Otherwise, high-type seller would rather hold on to the asset. Thus, when the unsophisticated buyers are willing to offer pooling price $C_H$ for sure, then the low-type seller has incentives to deviate and only trades with unsophisticated buyers in the market, therefore, efficient trading is not feasible. One implication of for Lemma 4 is the presence of sophisticated investors may deteriorate the trading efficiency.

# 4 Stationary Equilibrium

In this section, we construct a stationary equilibrium. In the stationary equilibrium, the strategies for sellers and buyers are independent of calendar time, that is $\sigma_t^z = \sigma^z$ for $z \in \{S, U\}$ and $\mu_t^\theta = \mu^\theta$ for $\theta \in \{H, L\}$. In the following lemma, we derive the necessary conditions for the stationary equilibrium.

**Lemma 5.** *Under the stationary equilibrium,* $\pi_t = \pi^* \equiv \frac{C_H - V_L}{V_H - V_L}$

Note that $\pi^*$ is the threshold belief such that the unsophisticated buyers are break-even when they offer a pooling price $C_H$. Lemma 5 says that in the stationary equilibrium, the market belief is constant over time, and no trading does not reveal any information about asset quality in stationary equilibrium.

**Lemma 6.** *The continuation value for the low-type seller* $W^L = V_L$ *in the stationary equilibrium.*

Lemma 6 states that the continuation value for the low-type seller equals the buyer's valuation for the low-quality asset. Any continuation value different from $V_L$ is not consistent with the definition of stationary equilibrium. First, let's consider that $W^L < V_L$, in this case, the low-type asset is traded efficiently. From the Lemma 4, we know that it should be the case that $\lambda^H < \lambda^L = \lambda$. Therefore, conditional on trading not occurs, the market belief improves over time, which is not consistent with Lemma 5, which states that in stationary equilibrium, the market belief is constant over time. Similarly, we can argue that $W^L > V_L$ is not feasible in the stationary equilibrium using the same logic. Therefore, the continuation value for the low-type seller is uniquely pinned down. Given above lemmas, we can characterize the stationary equilibrium.

**Theorem 1.** *There is an unique stationary equilibrium which is consist of* $\{\pi^*, \sigma^{U*}, \mu^{L*}\}$ *such that*

$$\pi^* = \frac{C_H - V_L}{V_H - V_L} \tag{20}$$

*and unsophisticated offers*

$$\sigma^{U*}(C_H) = \frac{\gamma^*}{\lambda(1-m)} \tag{21}$$

$$\sigma^{U*}(V_L) = 1 - \frac{\gamma^*}{\lambda(1-m)} \tag{22}$$

*and low-type seller accepts*

$$\mu^{L*}(V_L) = \frac{\lambda m}{\lambda - \gamma^*} \tag{23}$$

*where $\gamma^*$ is the probability the low-type seller receives pooling offer in stationary equilibrium.*

In Theorem 1, I construct a stationary equilibrium, where unsophisticated buyers are randomizing between pooling offer $C_H$ and separating offer $V_L$, and low-type seller is mixing between accepting and rejecting $V_L$. One way to think about the mixing strategy for the low-type seller is that, ideally, she can always trade with sophisticated buyer at separating price, and only accepting pooling price from the unsophisticated buyers, thus, we can reach the stationary equilibrium in which both types of asset are traded at the same rate such that market belief is freezing. But the buyer's type is not observed, therefore, she cannot tell whether the separating price is from sophisticated buyer or unsophisticated buyer. The denominator $\lambda - \gamma^*$ in (23) captures the probability that low-type seller receives separating offer $V_L$ in the stationary equilibrium, and numerator $\lambda m$ captures the probability that sophisticated buyer offers $V_L$ to the low-type seller. For the low-type seller, he rejects separating offer with positive probability in order to freeze the market belief.

Given Theorem 1, we can have the the following result about the expected trading time in stationary equilibrium.

**Proposition 1.** *The expected trading time $\tau$ for both types of asset is given by*

$$\tau^* = \frac{1}{\lambda m + \gamma^*} \tag{24}$$

Apparently, the expected trading time in stationary equilibrium is decreasing with the share of informed buyers in market. To understand this result, I will break down the trading rate for the high quality asset $\lambda m + \gamma^*$ into two parts. First, $\lambda m$ in (24) captures the offer from the informed buyers, who are always willing to pay a high price $C_H$ for the high-quality asset, and this term is increasing in the share of informed buyers, since the more sophisticated buyers in the market, the more likely that she will receive a good price. Second, the term $\gamma^*$ captures the probability of pooling offer $C_H$ from the uninformed buyers. This term, however, doesn't depend on the share of informed buyers in the market.[14] Therefore, the expected trading time is decreasing with the share of informed buyers in the market.

# 5 Non-Stationary Equilibrium

In the previous section, we have characterized the unique stationary equilibrium. Stationary equilibrium requires that we fix the belief at level $\pi^*$ such that each agent repeats his strategy over time. In this section, I will characterize equilibrium of fully dynamic game with initial market belief about the asset quality given. To help us characterize the non-stationary equilibrium, we will first derive two lemmas about the trading dynamics.

**Lemma 7.** *(Cream-skimming) When $\pi_t < \pi^*, W_t^L < V_L$ and when $\pi_t > \pi^*, W_t^L > V_L$*

This is the key result for the equilibrium dynamics in non-stationary equilibrium. Lemma 7 states that when the market belief is above the stationary belief, the continuation value for low-type seller is above the buyers' valuation of low-quality asset, which leads to the trade break down between low-type seller and informed buyers in the market, even though positive gains from trade can be realized in the potential trade ($V_L - C_L > 0$), this creates the strategic delay incentives for the low-type seller when the market condition is favorable. On the other hand, when the market belief is below $\pi^*$, then the reservation value for seller with low-quality is below $V_L$. Intuitively, the market belief affects the bidding strategy for the uninformed investors in the market. In particular, when the market belief is very high, the uninformed investors are more likely to bid $C_H$ for the average asset. Anticipating that the uninformed buyers are willing to pay $C_H$, low-type seller has incentive to strategically reject any offer from informed buyers and waits to trade with uninformed buyers. Assumption 2 guarantees that the benefit from waiting outweighs the cost of waiting, therefore, the reservation value for the low-type seller is higher than buyers' valuation for low-quality asset. On the other hand, when the market belief is below $\pi^*$, the uninformed investors is pessimistic about the asset quality, and they are only willing to pay a fair price to buy the low quality asset. Following Lemma 7, we can show that the trading rate for the two types of asset are given by

**Corollary 1.** *When the market belief $\pi_t < \pi^*$, $\lambda_t^L = \lambda$, and $\lambda_t^H = \lambda s$; when the market belief $\pi_t > \pi^*$, the $\lambda_t^L = \lambda(1-s)$, and $\lambda_t^H = \lambda$*

In the model, the trading rate for each type of asset can be interpreted as the trading volume.

[14]The $\gamma^*$ is the probability that a pooling offer is submitted by unsophisticated buyer which is pinned down by the continuation value of low-type seller in equilibrium

Then Corollary 1 says that the trading volume for each type of assets depend on the market belief about the asset quality. When market belief is very low, low-quality assets (lemon) are traded more frequently, while when the market belief is very high, assets with high-quality asset are actively traded. The first part of Corollary 1 is consistent with Daley and Green (2012, 2016) where when the market is very pessimistic, high-type sellers are more inclined to wait, and therefore, the low-quality assets are more actively traded.

Given the Lemma 7 and Corollary 1, we can summarize the bidding behavior for the uninformed buyers, and the probability that the low-type seller receives a pooling offer $C_H$ is given by

$$\gamma_t = \begin{cases} 0 & \text{if } \pi_t < \pi^* \\ \gamma^* & \text{if } \pi_t = \pi^* \\ \lambda(1-s) & \text{if } \pi_t > \pi^* \end{cases}$$

we can characterize the continuation value for the low-quality seller, and therefore, characterize the full dynamics for the non-stationary equilibrium.

Recall that the continuation value dynamics follows (13), and from Lemma 7, we know the market dynamics follows

$$d\pi_t = \begin{cases} \lambda\,(1-s)\,\pi_t\,(1-\pi_t)\,dt & \text{if } \pi_t < \pi^* \\ -\lambda s \pi_t\,(1-\pi_t)\,dt & \text{if } \pi_t > \pi^* \end{cases} \tag{25}$$

with $\pi_0$ is given. In the non-stationary equilibrium, the market belief about the asset quality are monotonically increasing (decreasing) depending on the initial market belief. And in either cases, the belief converges to the stationary belief level. Therefore, we can define $t_i^*(i = 1, 2)$ as the time it takes to converge to stationary equilibrium when $\pi_0 < \pi^*$ $(i = 1)$ and $\pi_0 > \pi^*$ $(i = 2)$ And then

$$t_1^* = \frac{1}{\lambda\,(1-s)}\left(\log(\frac{\pi^*}{1-\pi^*}) - \log(\frac{\pi_0}{1-\pi_0})\right) \tag{26}$$

$$t_2^* = \frac{1}{\lambda s}\left(\log\left(\frac{\pi_0}{1-\pi_0}\right) - \log\left(\frac{\pi^*}{1-\pi^*}\right)\right) \tag{27}$$

The time it takes to converge the stationary belief depend on the initial belief $\pi_0$, and the learning speed $(\lambda_t^L - \lambda_t^H)$. And the relationship between share of informed buyers and learning speed is not monotonic

**Theorem 2.** *The equilibrium dynamics in the non-stationary game is characterized by* *(i) When $\pi_0 < \pi^*$, the equilibrium dynamics are given by*

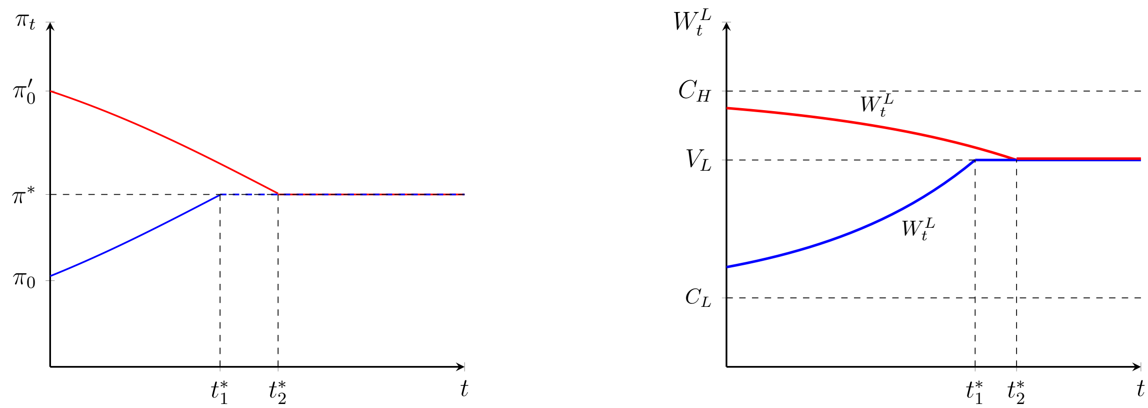
$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{-\lambda(1-s)t}} & t < t_1^* \\ \pi^*, \quad t \geq t_1^* \end{cases} \tag{28}$$

$$W_t^L = \begin{cases} C_L + (V_L - C_L)e^{-r(t_1^*-t)} & t < t_1^* \\ V_L \quad t > t_1^* \end{cases} \qquad v \tag{29}$$

*(ii) When $\pi_0 > \pi^*$, the equilibrium dynamics are given by*

$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{\lambda st}} & t < t_2^* \\ \pi^*, \quad t \geq t_2^* \end{cases} \tag{30}$$

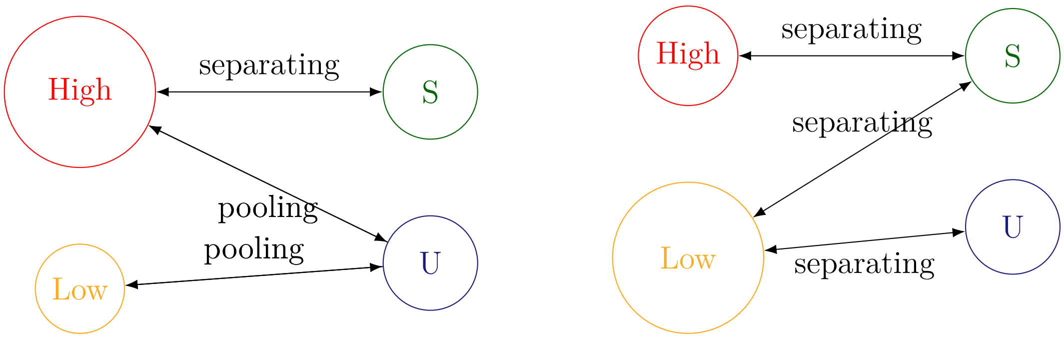
$$W_t^L = \begin{cases} V_L + \left(1 - e^{-(r+\lambda(1-s))(t_2^*-t)}\right)(C_H - V_L) & t < t_2^* \\ V_L \quad t > t_2^* \end{cases} \tag{31}$$




(a) Market Belief (b) Continuation Value

Figure 2: Non-stationary Equilibrium

Theorem 2 characterize the equilibrium dynamics with different initial market belief $\pi_0$. There are two regimes in the non-stationary equilibrium (see figure 3): when the initial belief is low, the market is pessimistic about the asset quality sold on the market, and then unsophisticated buyers are only willing to pay a separating price for the low-quality asset. For the low-type sellers, anticipating that pooling offer is not feasible in the market, they do not have an incentive to delay, therefore, they trade efficiently. For the sophisticated buyers, since they can identify the quality sold on the market, they always bid separating offer to the seller, and both types of sellers in the market are willing to take the offer from




(a) optimistic

(b) pessimistic

Figure 3: Trading pattern

the sophisticated buyers.

When the market is optimistic about the asset quality (see figure 3(a) , the unsophisticated buyers are willing to pay a pooling price for both types of assets in the market. Therefore, in this situation, the low-type sellers realize that they can get a better offer by waiting for pooling offer from the unsophisticated buyers, therefore, the low-type seller's waiting value is higher than buyers' valuation of low-quality asset, therefore, the trade between low-type seller and sophisticated buyers break down completely.

Market is partially segmented in both scenarios. When the market is optimistic, low-type seller only trades with unsophisticated buyers in the market due to the strategic incentive of waiting; when the market is pessimistic, high-type seller only trades with sophisticated buyers in the market, since the asymmetric information between them is not severe to break down to forgo the gains from trade.

Theorem 2 can also shed some light on the signaling role of time to sale. When the market is pessimistic about the asset quality, no trade is a good news to the investors, since the low-type sellers are more eager to trade in this region; when the market is optimistic about the asset quality, no trade is a bad news to the investors, since only low-type sellers are eager to delay and wait for unsophisticated buyers. In the stationary equilibrium, however, time to sale does not transmit any information about the asset quality.

**Proposition 2.** *The average trading time for both types of seller are given by* *(i) When $\pi_0 < \pi^*$*

$$\tau_H = \frac{1}{\lambda s} + e^{-\lambda s t_1^*}\left(\frac{1}{\lambda s + \gamma^*} - \frac{1}{\lambda s}\right) \tag{32}$$

$$\tau_L = \frac{1}{\lambda} + e^{-\lambda t_1^*}\left(\frac{1}{\lambda s + \gamma^*} - \frac{1}{\lambda}\right) \tag{33}$$

*(ii) When $\pi_0 > \pi^*$*

$$\tau_H = \frac{1}{\lambda} + e^{-\lambda t_2^*}\left(\frac{1}{\lambda s + \gamma^*} - \frac{1}{\lambda}\right) \tag{34}$$

$$\tau_L = \frac{1}{\lambda(1-s)} + e^{-\lambda s t_2^*}\left(\frac{1}{\lambda s + \gamma^*} - \frac{1}{\lambda(1-s)}\right) \tag{35}$$

*where $t_1^*, t_2^*$ are given by (26) and (27).*

Proposition 2 shows the average trading time for two types of sellers given different initial market belief. There are two components in the average trading time for the sellers: the first component is the how long does is take to converge to stationary equilibrium, the second component is the average trading time in stationary equilibrium. When the market belief is low, the low-type seller trades faster along the non-stationary equilibrium path, but the probability that the equilibrium converges to the stationary equilibrium is lower. Therefore, the expected trading time for the low-quality asset is lower. We can get the opposite result when the market belief about the asset quality is high.

Following the above analysis, I can compare the expected trading time for each type of asset.

**Corollary 2.** *The average trading time for both types of seller has the following relation: when $\pi_0 < \pi^*, \tau_H < \tau_L$; when $\pi_0 > \pi^*, \tau_H > \tau_L$.*

This is a direct result from Proposition 2. When $\pi_0 < \pi^*$, the average trading time for low-quality asset before reaching the stationary equilibrium is lower, and probability to reach the stationary equilibrium is also lower. Since the trading rate for low-quality asset in stationary equilibrium is lower than that before converging to stationary equilibrium, it is easy to find that the average trading time for low-quality asset is lower. Similarly, we can get the opposite result when $\pi_0 > \pi^*$

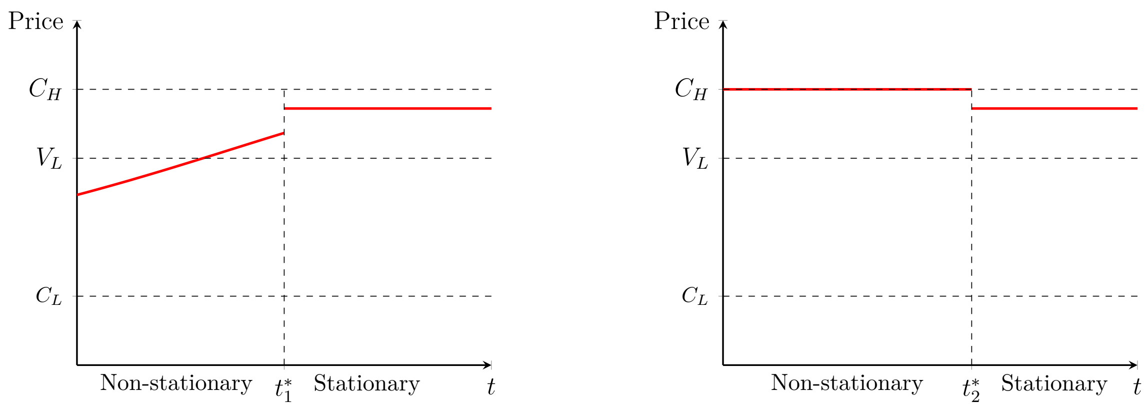
## 5.1 Averaging Trading Price

In this subsection, we consider the relationship between average trading price and time-on-the-market. We define the average trading price $AP_t$ at time $t$ as

$$AP_t = \begin{cases} \frac{\pi_t s C_H + (1-\pi_t) W_t^L}{\pi_t s + 1 - \pi_t} & \pi_t < \pi^* \\ \frac{(\pi^* s + \frac{\gamma^*}{\lambda}) C_H + (1-\pi^*) s V_L}{s + \frac{\gamma^*}{\lambda}} & \pi_t = \pi^* \\ C_H & \pi_t > \pi^* \end{cases} \tag{36}$$

where $m\pi_t + (1 - \pi_t)$ is the total probability that the asset can be traded at time $t$ if the seller meets a buyer. And with probability $\frac{s\pi_t}{s\pi_t + (1-\pi_t)}$ the asset is traded between high-type seller and sophisticated buyers at price $C_H$; with probability $\frac{(1-\pi_t)}{s\pi_t + (1-\pi_t)}$ the trade is between low-type seller and both types of buyers at reservation price $W_t^L$. When the initial belief about the asset quality is high, low-type seller values more by holding the asset, thus, trade only occurs at price $C_H$.

After characterize the trading price on the equilibrium path, we can show the trading price and time-on-the-market in Figure 3. In the first scenario, when the initial quality of asset pool is low, trading price is increasing with time-on-the-market until it reaches the stationary equilibrium. In stationary equilibrium, there is a jump in the asset price, and this is because that in stationary equilibrium, unsophisticated




(a) $\pi_0 < \pi^*$ (b) $\pi_0 > \pi^*$

Figure 4: Average Trading Price

## 5.2 Market liquidity

In this section, we consider the market liquidity dynamics. In the model, I define the market liquidity as the average trading rate for the asset (trading volume) The expected trading rate for the asset in the market is given by

$$\eta_t = \pi_t \lambda_t^H + (1 - \pi_t)\,\lambda_t^L \tag{37}$$

$$= \lambda_t^L + \pi_t(\lambda_t^L - \lambda_t^H) \tag{38}$$

From (38), we can see that the market liquidity depends on the market belief ($\pi_t$) and the relative trading rate for the two types of asset ($\lambda_t^L - \lambda_t^H$). Note that higher market belief not necessarily leads to higher market belief,

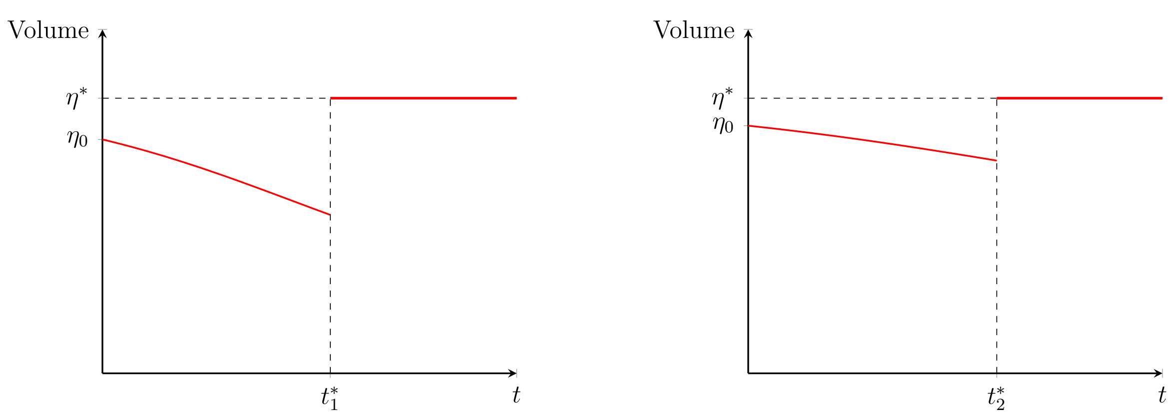
**Proposition 3.** *The expected trading rate for the asset is given by*

1. *when* $\pi_0 < \pi^*$,

$$\eta_t = \begin{cases} \lambda - \dfrac{\pi_0 \lambda (1-s)}{\pi_0 + (1-\pi_0) e^{-\lambda(1-s)t}} & \text{for } t < t_1^* \\ \gamma^* & \text{for } t \geq t_1^* \end{cases} \tag{39}$$

2. *When* $\pi_0 > \pi^*$,

$$\eta_t = \begin{cases} \lambda(1-s) + \dfrac{\pi_0 \lambda s}{\pi_0 + (1-\pi_0) e^{\lambda s t}} & \text{for } t < t_2^* \\ \gamma^* & \text{for } t \geq t_2^* \end{cases} \tag{40}$$




(a) $\pi_0 < \pi^*$ (b) $\pi_0 > \pi^*$

Figure 5: Trading volume

In Proposition 3, the market liquidity (trading volume) is decreasing over time regardless of the initial belief level. And then it bounces back once the game is reaching the stationary equilibrium. The intuition for this result is as follows: when the initial belief is very low, market infers the asset quality based on the time on the market, asset with high-quality is more likely to stay in the market, since it only trades with sophisticated buyers. But since assets with low-quality are traded faster than those of high quality in this region, therefore, the market liquidity of asset decreases due to the inference based on time-on-the-market. Once the game reaches the stationary equilibrium, the unsophisticated buyers are willing to offer pooling price $C_H$, which leads to a jump in the average trading rate. The underlying mechanism that drives this result is that the trading rates of both types of asset not only determines the belief updating for the uninformed buyers, but also contributes the expected trading volume.

This section, we characterize the payoff to uninformed buyers. The cream-skimming literature predicts that due to the information advantage from the sophisticated investors, which generate negative externality on the unsophisticated buyers.

The expected payoff to the unsophisticated buyer is given by

$$\Pi^U = \begin{cases} (1-\pi_t)(V_L - W_t^L) & \text{if } \pi_t < \pi^* \\ \pi_t(V_H - C_H) + (1-\pi_t)(V_L - C_H) & \text{if } \pi_t > \pi^* \end{cases} \tag{41}$$

And the expected payoff to the sophisticated buyer is given by

$$\Pi^S = \begin{cases} \pi_t(V_H - C_H) + (1-\pi_t)(V_L - W_t^L) & \text{if } \pi_t < \pi^* \\ \pi_t(V_H - C_H) & \text{if } \pi_t > \pi^* \end{cases} \tag{42}$$

# 6 Comparative Statics

I focus on two parameters of interest: the share of sophisticated investors in the market and the search friction. In the context of the model, both parameters

## 6.1 Share of sophisticated buyers

In the model, the share of sophisticated buyers plays a key role. First, it can shape the equilibrium structure. From Assumption 2, the share of sophisticated buyers ($s$) is crucial, and it affects the outside option for the low-type. When the ratio of sophisticated buyers is very high, then on average, it takes longer for the low-type seller to meet with the unsophisticated buyers, and therefore, the continuation value is lower given for any given time to sale. Thus, due to the high search cost for the low-type seller, she is willing to take separating offer from the buyer, and in this case the trade is efficient. Second, it can facilitate learning for the unsophisticated buyers. Since the unsophisticated buyers infer quality through the time-to-sale, and high-type seller is willing to trade with sophisticated buyers for sure. Then the share of sophisticated buyers affect the learning speed for the unsophisticated buyers.

In this section, I will consider how does the share of informed buyers in the market affect the market liquidity, and trading volumes.

**Proposition 4.** *When $\pi_0 < \pi^*$, $\frac{d\tau_L}{ds} < 0$, $\frac{d\tau_H}{ds}$ is undetermined; When $\pi_0 > \pi^*$, $\frac{d\tau_H}{ds} < 0$, $\frac{d\tau_L}{ds}$ is undetermined.*

Figure 2 shows the relationship between the share of informed buyer and expected trading time. There are two points need to be emphasized in this figure. First, it takes longer for the low-quality asset to be traded when the initial market belief about the asset quality is low. Second, the expected trading time is not monotone in the ratio of informed buyers in the market. We can interpret the ratio of informed buyers in the market as the market transparency, Figure 2 shows that when market belief $\pi_0 > \pi^*$, the relationship between average trading time and ratio of informed buyers is U-shape, which means that by increasing market transparency, the average trading time for low-quality seller could increase.

When there are more sophisticated buyers in the market, then it seems that the low-quality seller has less incentive to delay trade, since they are less likely to meet an uninformed buyer. But when there are more sophisticated buyers in the market, the learning effect improves, and it takes less time to reach the stationary equilibrium, moreover, the trading rate in stationary equilibrium is higher, hence the overall effect on expected trading rate is ambiguous, and this is why there are more informed buyers in the market, the expected trading time for low-quality seller could increase.

## 6.2 Search Friction

In the main model, I assume that the search intensity is independent of seller's type for simplicity, in this section, I allow type dependent search intensity. Specifically, assume that the search intensity for type-$\theta$ seller is $\lambda_\theta$ for $\theta \in \{H, L\}$. [15]. Without abuse of notation, I continue to use $\lambda_t^\theta$ as the trading rate for the type-$\theta$ asset. In order to get rid of the trivial case, we assume that

[15] I do not need to require that $\lambda_H > \lambda_L$ or $\lambda_H < \lambda_L$, either case can hold in this extension

**Assumption 3.** $\lambda_L > \lambda_H m$

This assumption states that the search intensity for low-type seller is greater than the search intensity of high-type only targeting sophisticated buyers in the market. This assumption is aim to get rid of the trivial belief dynamics where the market belief and time-on-the market is purely driven by the search friction. If this assumption does not hold, then the market belief decreases regardless of the strategy taken by the sellers.

Then under Assumption 3, we can get the qualitatively the same result. The key insight is that when the buyers make take-it-or-leave offer, the search intensity for high-type seller does not play too much role in the equilibrium, and it only affects the learning speed for the unsophisticated buyers.

# 7 Extensions

In the main model, I make some assumptions about the ratio of sophisticated investors and the composition of asset quality. In this extension, I consider the equilibrium without Assumption 2, and discuss how does it change the equilibrium dynamics. And then I introduce the asset (skill) depreciation into the model, and show that the result is still robust under mild asset depreciation.

## 7.1 Equilibrium without Assumption 2

In this section, we characterize the equilibrium when Assumption 2 does not hold. In this case, the strategic delay incentives for the low-type seller disappear due to the waiting cost is so high such that the payoff from targeting only the uninformed buyer in the market cannot compensate the cost of waiting.

**Lemma 8.** *The continuation value for the low-type seller in stationary equilibrium is*

$$W^L = \frac{rC_L + \lambda(1-s)C_H}{r + \lambda(1-s)} \tag{43}$$

And when Assumption 2 doesn't hold, then we know that $W^L < V_L$. Define $\pi^\dagger$ as the threshold belief such that uninformed buyer is indifferent between offering $C_H$ and $W^L$, then

$$\pi_t(V_H - C_H) + (1-\pi_t)(V_L - C_H) = (1-\pi_t)(V_L - W^L) \tag{44}$$

which gives

$$\pi^\dagger = \frac{C_H - W^L}{V_H - W^L} > \pi^* \tag{45}$$

Then we can characterize the equilibrium in the following proposition.

**Proposition 5.** *When Assumption 2 does not hold, the equilibrium is characterized by*

*(i) when $\pi_0 < \pi_\dagger$, the equilibrium is characterize by $\{\pi_t, W_t^L\}$ such that*

$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{-\lambda(1-s)t}} & t < t^\dagger \\ \pi^\dagger & t \geq t^\dagger \end{cases} \tag{46}$$

*where $t^\dagger$ is given by*

$$t^\dagger = \frac{1}{\lambda(1-s)}\left(\log(\frac{\pi^\dagger}{1-\pi^\dagger}) - \log(\frac{\pi_0}{1-\pi_0})\right) \tag{47}$$

*(ii) when $\pi_0 < \pi_\dagger$,*

$$W_t^L = W^L$$

$$\pi_t = \pi_0$$

*where uninformed buyers are bidding $C_H$ with probability one, and informed buyers bid $C_H$ for high-quality asset and $W^L$ for low-quality asset.*

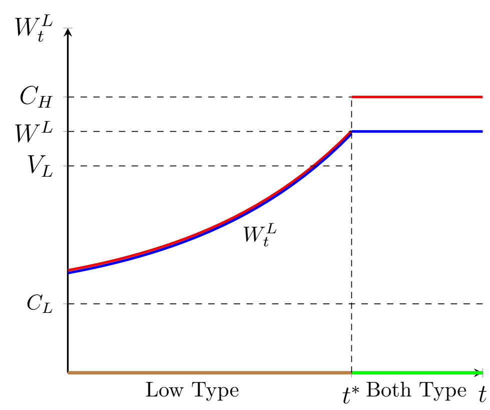



Figure 6: Equilibrium without Assumption 2

## 7.2 Quality Shocks

One argument in In the main model, I assume that the quality of the asset is fixed over time, in this extension, I introduce the quality shocks to the asset. In particular, for high type asset, it receives a quality shock with intensity $\beta$ which turns the high-quality asset into a low-quality asset. Assume that the quality shock is private information to the high-quality seller, and is not observable to the buyers, regardless of sophisticated buyers or unsophisticated buyers. In this case, I can define the holding value for the high-quality asset as $C'_H$, which is given by

$$C'_H = \frac{rC_H + \beta C_L}{r + \beta} \tag{48}$$

and the higher the quality shock $\beta$, the lower the holding value for high type asset. Similarly, the holding value for type-H asset is

$$V'_H = \frac{rV_H + \beta V_L}{r + \beta} \tag{49}$$

Given that the asset has been sold before time $t$, market (unsophisticated buyers) update the belief as follows

$$\pi_{t+\mathrm{d}t} = \frac{\pi_t e^{-(\lambda_t^H + \beta)\,\mathrm{d}t}}{\pi_t e^{-\lambda_t^H\,\mathrm{d}t} + (1 - \pi_t)e^{-\lambda_t^L\,\mathrm{d}t}} \tag{50}$$

where $\lambda_t^\theta$ is the trading rate for type-$\theta$ asset. The denominator in (50) is the probability that it is of high quality, conditional on that it is high quality at time $t$, while the denominator is the probability that no trade between $[t, t+\mathrm{d}t]$. I can rewrite the belief dynamics as follows:

$$\frac{\mathrm{d}\pi_t}{\mathrm{d}t} = \pi_t(1 - \pi_t)(\lambda_t^L - \lambda_t^H) - \beta\pi_t \tag{51}$$

where the first term on the RHS of (51) captures learning through relative trading rate, and the second term captures the quality shock.

**Assumption 4.** *The quality shock intensity satisfies*

$$\beta < \lambda(1 - s) \tag{52}$$

Assumption 4 provides a upper bound on the quality shock intensity to the seller. Without this assumption, the belief dynamic is strictly decreasing over time, since the quality shock dominates the learning effect.

Denote the continuation value for the high-$\theta$ seller as $W_t^\theta$ and then

$$rW_t^H = c_H + \beta(W_t^L - W_t^H) + \frac{\mathrm{d}W_t^H}{\mathrm{d}t} \tag{53}$$

where $\beta(W_t^L - W_t^H)$ captures the quality shock to the high-type seller. Note that the continuation value for the high-type seller could be higher than her holding value. This is the case if the unsophisticated buyers are inclined to pay a pooling price and the continuation value for the low-type is higher than unsophisticated buyer's valuation for the low-quality asset.

For the low-type seller, her continuation value is as follows

$$rW_t^L = c_L + \gamma_t(W_t^H - W_t^L) + \frac{\mathrm{d}W_t^L}{\mathrm{d}t} \tag{54}$$

where $\gamma_t$ is the probability that pooling offer is made by the unsophisticated buyers. Since the buyer has the bargaining power, the pooling offer is the high-type seller's continuation value. We can rewrite the ODE system as follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} W_t^H \\ W_t^L \end{pmatrix} = -\begin{pmatrix} c_H \\ c_L \end{pmatrix} + \begin{pmatrix} r+\beta & -\beta \\ -\gamma_t & r+\gamma_t \end{pmatrix}\begin{pmatrix} W_t^H \\ W_t^L \end{pmatrix} \tag{55}$$

#### 7.2.1 Stationary Equilibrium

Denote $\pi^\star$ as the stationary equilibrium belief. At the stationary belief, the unsophisticated buyers are indifferent between pooling offer and separating offer. That is

$$\pi^\star(V_H' - W^{H\star}) + (1-\pi^\star)(V_L - W^{H\star}) = 0 \tag{56}$$

At the stationary equilibrium, the continuation value for the low-type is

$$W^{H\star} = \frac{rC_H + \beta V_L}{r+\beta} \tag{57}$$

$$W^{L\star} = V_L \tag{58}$$

Therefore, the stationary belief is the same $\pi^\star = \pi^*$ under no quality shock. Then I can characterize the stationary equilibrium as follows.

**Theorem 3.** *When Assumption* **??** *holds, there exist a stationary equilibrium where*

1. *the unsophisticated buyers randomize between pooling offer* $W^{H\star}$*and separating offer*

$W^{L\star}$

2. *Given the separating offer* $W^{L\star}$*, low-type sellers accept it with probability* $\mu^L(W^{L\star})$

3. *The market belief stays at* $\pi^\star$ *conditional there is no trading.*

The main difference with the benchmark model is that with quality shock, the trading rates for two types of the assets are not the same. In the benchmark model, the low-type seller turns down some separating offer to freeze the market belief where the unsophisticated buyers are willing to pay a pooling price. In the model with quality shock, the quality of the asset deteriorates over time, in order to maintain the stationary belief, the low-type seller trades faster to compensate the belief decreasing due to the quality shock.

# 8 Empirical Implication

Following the model, we can derive several empirical implications to test.

In the context of IPOs, Ritter and Welch (2002) write that "it is conventional wisdom among both academics and practitioners that the quality of firms going public deteriorates as a period of high issuing volume progresses." In terms of my model, it means that the average trading rate is positive correlated with the market belief about asset quality. First, notice that two regimes exist in the model that have distinct predictions on the role of delayed trade, and both of them are plausible, therefore, to test the result of whether trade delay is positive correlated with the post-performance or price, one need to control for the different regime. In the model, the two regimes are separated by the market belief, which is the cut-off of the average quality of asset in the market. In the IPO market, if we want to separate the two regimes, one way is to consider hot market [16] and cold market separately, and test whether in hot market, the time-to-IPO is negative correlated with post-performance. Similarly, in cold market, we can test whether time-to-IPO is positive correlated with post-performance.

Real estate market is a more suitable place to test the my model with two regimes. We can interpret the initial quality about the asset as the neighborhood house quality, and then we can test the relationship between time-on-the-market and asset prices and trading volume in different neighborhood with different quality. In the model, I predict that house in better neighbourhood corresponds to the case with optimistic belief, therefore, the longer time-on-the-market should be associated with lower trading price, On the contrary, for houses in

[16] In the IPO literature, hot market is defined as the IPO volume and the ratio of under-pricing, in my model, the hot market is defined as the average quality about firm to go to IPO. In general, those firms who chooses to go IPO are endogenous, so the average quality for those IPO firms are also endogenous

worse neighbour quality, we should predict that the time on the market is positive correlated with sales price.

Another potential test is to consider the time to sale in normal times versus during crisis. In normal times, the perceived asset quality sold on the market is high, therefore, waiting is not too costly for the low-type seller. In crisis, buyers on the market are pessimistic about the asset quality, therefore we can test whether time-to-sale is positively correlated with asset quality during crisis, and negatively correlated with asset quality during normal times.

Another set of test is to test how does the share of sophisticated investors affect the expected trading time. We can interpret the informed buyers in the real market as dealers, and in general, they are better informed than retail investors. One way to test the relationship is to consider how does the number of dealers in specific region will affect the average trading time of the real estate. In the model, we find that the relationship between the ratio of informed buyers and expected trading time is not monotonic, in this setting we can test whether number of dealers will affect the average trading time of the real estate.

Most importantly, Assumption 2 gives us the condition of the cream-skimming effect. One implication for Assumption 2 is that, in a market with less search friction (higher $\lambda$), and less fraction of sophisticated investors, the cream-skimming effect is more likely to exist. Therefore, I can test the time-to-sale in markets with different search friction, and whether in market with less search friction

# 9 Conclusion

In this paper, I build a dynamic model to reconcile the two contradicting predictions of delayed trade on performance (price). In the model, a seller with one unit asset to sell, and asset quality could be either high or low, which is his private information. Two types of buyers in the market, informed buyers know perfectly about the asset quality, uninformed buyers know nothing, and only observe how long the seller has been in the market. Seller searches buyers in the market, after they meet buyers make take-it-or-leave-it offers to sellers. There are two regimes in the model, when the market is very low, low-quality seller has no incentives to delay trading, but for high-quality seller, she is only willing to trade with informed seller. When the market belief is very high, since the uninformed buyers are willing to make a high pooling offer to both types of asset, then low-quality seller would like to wait for uninformed buyers instead of trading with informed seller.

The model is consistent with both the good signal story and bad signal story, and it depends on which type of seller wants to delay trading. When the market belief is very low, it is the high-quality seller that wants to delay trading since the offer made by the uninformed buyers is so low that he prefers to delay take the outside option, hence in this case, delayed trade is a good signal about the quality. When the market is very high, it is the low-quality seller who wants to delay trading, because they know that in good market condition, even the uninformed buyers in the market are willing to offer high price to the seller.

The model has the following predictions. First, it turns out when the market belief is below the stationary belief, low-quality asset trades fast and the expected trading time for low-quality asset is lower. Similarly, when the market belief is above the stationary belief, the high-quality asset trades at a higher rate, and hence the expected trading time for high-quality asset is lower. Another prediction of the model is that when the market belief is below the stationary belief, trading volume is decreasing with the market belief, when the market belief is above the stationary belief, the trading volume is increasing with the market belief. But when the market belief is above the stationary belief, the trading price is not sensitive to how long the seller has been on the market except that the equilibrium converges to the stationary equilibrium. This result is due to the fact that when the market belief is very high, only pooling offer could lead to transaction, hence the price is not sensitive to market belief in this region.

The model can also shed light on the relationship between market transparency and expected trading time (liquidity). In the model, the ratio of informed buyers in the market can be interpreted as market transparency. It turns out that the expected trading time of sellers is not always monotonic with market transparency. The ratio of informed buyers in the market affects the expected trading time through three channels, the first is that it affects the trading rate before the equilibrium converges to stationary equilibrium. The second is that it affects the probability that the equilibrium will converge to stationary equilibrium. The third one is that it affects the trading rate in stationary equilibrium. Due to these three effects altogether, the ratio of informed buyers and expected trading time is not always monotonic.

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# Appendix A

## Proof of Lemma 1

This is direct result from (9). When the price is higher than the continuation value $W_t^\theta$, the optimal strategy for the seller is to accept offer, when the price is below the continuation value $W_t^\theta$, then the optimal strategy is to reject the offer.

## Proof of Lemma 2

Since there are only two types of asset in our model, given any price p offered by the unsophisticated investors, the payoff is given by

$$\pi_t \left(V_H - p\right) \mu_t^H \left(p\right) + \left(1 - \pi_t\right) \left(V_L - p\right) \mu_t^L \left(p\right) \tag{59}$$

where $\mu_t^H = 0$ for $p < C_H$ First, we show that any price $p \in \left(p_t^l, p_t^h\right)$ is not optimal. To see this ,when $p \in \left(p_t^l, p_t^h\right), \mu_t^L \left(p\right) = 1$, then by reducing the price offer from p to $p - \varepsilon$ such that $p - \varepsilon \in \left(p_t^l, p_t^h\right)$, this offer is still accepted by low-type seller for sure, but it can improve the profit to the uninformed buyer.

Second, we show that any price $p_t > C_H$ is not optimal. This is trivial, since the uninformed buyer could always lowers $p_t$ a little bit which doesn't affect the decision for the sellers but improve the profit to the uninformed buyer.

Third, we want to show that non-serious offer could be made by the uninformed buyers. This is indeed the case when

$$\pi_t(V_H - p^h) + (1 - \pi_t)(V_L - p^h) < 0 \tag{60}$$

and

$$W_t^L < V_L \tag{61}$$

where (60) states that offer $p^h$ is not profitable since it generates negative profit, and (60) states that $p_l$ is also not optimal, since the price paid to the low-quality asset is higher than the value to the uninformed buyers. Thus, in this case, only non-serious offers are made.

## Proof of Lemma 4

We prove Lemma 4 by contradiction. Suppose that under Assumption 1 there exist an equilibrium where $\lambda_t^H = \lambda_t^L = \lambda$, that is both types of asset are traded at efficient rate.

Then going forward, conditional on there is no trading occur, both types of sellers and buyer will repeat their strategies at time t , and this is due to the fact that the market belief stays constant, and the bidding strategy for the buyers are unaffected. For $\lambda_t^H = \lambda$, it requires that $\sigma_t^U(C_H) = 1$, otherwise, the high-type seller can always hold on to the asset instead of trading with the uninformed buyer. Similarly, in order to have $\lambda_t^L = \lambda$, it requires that $p_l = W_t^L \leq V_L$, and $\mu_t^L(p_l) = 1$. Therefore, for $t' > t, \sigma_t^U(C_H) = 1$ and $\sigma_t^I(C_H) = \mathbb{I}(\theta = H), \sigma_t^I\left(W_t^H\right) = \mathbb{I}(\theta = L)$

Under this strategy, the continuation value to the low-quality seller is

$$W_t^L \leq \frac{rC_L + \lambda\left(mV_L + (1-m)\, C_H\right)}{r+\lambda}$$

and the inequality holds when $W_t^L = V_L$. But from Assumption 1, we know that

$$V_L < \frac{rC_L + \lambda\left(mV_L + (1-m)\, C_H\right)}{r+\lambda}$$

which contradicts with $p_l = W_t^L \leq V_L$. Therefore, equilibrium with $\lambda_t^H = \lambda_t^L = \lambda$ is not feasible under Assumption 1.

## Proof of Lemma 3

*Proof.* Given the market belief $\pi_t$, only two prices are offered with positive probability, either $C_H$ or $\min\left\{V_L, W_t^L\right\}$, the payoff from offering $C_H$ is

$$\pi(t)\left(V_H - \alpha V_H\right) + (1-\pi(t))\left(R_L(t) - \alpha V_H\right) \tag{62}$$

and the payoff from offering $\min\left\{V_L, W_t^L\right\}$ is given by

$$(1-\pi_t)\left(V_L - \min\left\{V_L, R_L(t)\right\}\right) \tag{63}$$

So comparing (62) and (63), it is easy to see that when $\pi(t) > \frac{C_H - \min\{R_L(t), V_L\}}{V_H - \min\{R_L(t), V_L\}}$,uninformed buyers are willing to offer $\alpha V_H$, and when $\pi_t < \frac{\alpha V_H - \min\{R_L(t), V_L\}}{V_H - \min\{R_L(t), V_L\}}$, the maximum price they are willing to offer is $V_L$ □

## Proof of Lemma 5

*Proof.* There are two parts in this lemma. Denote $W^{L*}$ as the continuation value for low-type seller in stationary equilibrium. First I will show that in stationary equilibrium, the continuation value for low-quality seller is $V_L$.

Suppose not, if $W^{L*} > V_L$, then low-quality seller will turn down any offer from informed buyers, since $V_L$ is the maximum that the informed sellers are willing to pay for the low-quality asset. But high-quality seller always trades with the informed buyer, hence the trading rate for high quality asset is higher $\lambda_H > \lambda_L$, which is against the Lemma 2. If $F_L^* < V_L$, then low-quality seller will trade for sure, since any price $W^{L*} + \varepsilon$ is sufficient to attract all the low-quality seller. But this is also against the Lemma 2. Thus, we can conclude that in stationary equilibrium, the continuation value for low-type seller should be $V_L$. □

## Proof of Proposition 1

*Proof.* From Lemma 5 we know that the reservation value for low-quality seller is $V_L$, thus he is indifferent between accepting and rejecting $V_L$. Similarly, in stationary equilibrium, the uninformed buyer is mixing between $V_L$ and $C_H$. According to Lemma 3, we know that $\pi^* = \frac{C_H - V_L}{V_H - V_L}$. Now we need to pin down the exact $\gamma_t$ in equilibrium. Recall from (13), in stationary equilibrium, we have

$$rW^{L*} = rC_L + \gamma^*(C_H - W^{L*})$$

which can uniquely pin down the $\gamma^*$ as $\gamma^* = \frac{r(V_L - C_L)}{C_H - V_L}$, and therefore the probability that the uninformed buyer offers $C_H$ is given by

$$\sigma^{U*}(C_H) = \frac{\gamma^*}{(1-m)\lambda}$$

And it is easy to check that $\sigma^{U*}(C_H) \in (0, 1)$ from Assumption 2. The remaining part is to pin down the probability that low-quality seller is willing to accept $V_L$. This can be solved based on Lemma 4, since both type of sellers are trading at the same rate, and given $\gamma^*$, we know that high-quality asset is traded at a rate $\lambda^H = m + (1-m)\gamma^*$. Hence the law-quality seller should also trade at rate $\lambda^L = \lambda^H$. And we can express the trading rate for the low-quality seller as:

$$\lambda^L = \lambda\left(\mu^{L*}(V_L)(m + (1-m)\sigma^U(V_L)) + (1-m)\sigma^U(C_H)\right)$$

Where the first term on RHS is the probability to accept offer $V_L$, and second term on the

RHS is the probability to trade at price $C_H$. Hence, we can get that the accepting rate for $V_L$ is given by

$$\mu^{L*}(V_L) = \frac{\mu}{1-(1-\mu)\sigma^{U*}(C_H)} \tag{64}$$
$$= \frac{\lambda m}{\lambda - \gamma^*} \tag{65}$$

□

## Proof of Lemma 6

*Proof.* There are two parts for this Lemma.

(i) I will show that when $\pi_t > \pi^*$, then $W_t^L > V_L$. Suppose not, there exists a $\pi' > \pi^*$ such that $W_t^L(\pi') \leq V_L$. Now I consider the first case where $F_t < V_L$. Then low-quality seller will trade for sure, hence $\lambda_L = \lambda$. However, from 4we know that we cannot have equilibrium where both type of sellers trade at rate $\lambda$, then it follows that $\lambda_H < \lambda_L$, so based on the belief dynamics **??**, the market belief goes up.

I claim that there exists a $\pi_1 > \pi'$ such that $W_t^L(\pi_1) = V_L$. Suppose this is not true, due to the continuity of $W_t^L$, we can conclude that for all $\pi_t > \pi'$, we have $W_t^L < V_L$, then applying the same logic, we can see that the trading rate for low-quality seller is always $\lambda_L = \lambda$, then the market belief goes up for all $\pi_t > \pi'$, but since the market belief is bounded above by 1, then $\pi_t$ will converge to $\pi_\infty < 1$, but since $\pi_\infty > \pi^*$, this cannot be as stationary belief, hence this is a contradiction.

Given that at $\pi_1 > \pi^*$, $W_t^L(\pi_1) = V_L$, and for all the $\pi_t \in (\pi', \pi_1)$, $R_t < V_L$, now consider $\pi_t'' = \pi_1 - \varepsilon$, where $\varepsilon$ is small enough, then we know that $W_t^L(\pi_t'') \to V_L$ since reservation value is continuous in $\pi_t$, then we know that at $\pi_t''$, uninformed buyers are willing to offer $C_H$ since by offering $W_t^L$, then can only get a payoff close to zero. Hence at $\pi_t''$, the both high-quality seller and low-quality seller will trade at rate $\lambda$, which is against Lemma 4.

Now suppose that there exists a $\pi_t$ such that $W_t^L(\pi_t') = V_L$, then for all $\pi_t \in (\pi^*, \pi_t')$ we have $W_t^L > V_L$, now consider at the $\pi_t'$, the dynamics of continuation value should be

$$rW_t^L = rC_L + \gamma(C_H - W_t^L) + \frac{\mathrm{d}W_t^L}{\mathrm{d}t}$$

Since at$\pi'_t$, unsophisticated buyers offer $C_H$, then high-quality seller trades at maximum rate $\lambda$, then the market belief goes down, hence $\pi'_{t+\mathrm{d}t} < \pi'_t$, so we can see that $\frac{\mathrm{d}W_t^L}{\mathrm{d}t} > 0$, also $\gamma = \lambda(1-s)$ at $\pi'$, then we can rewrite the continuation value of the low-quality seller at $\pi'$ as

$$W_t^L = \frac{rC_L + \lambda(1-\mu)C_H + \frac{\mathrm{d}W_t^L}{\mathrm{d}t}}{r + \lambda(1-s)}$$

which should be greater than $V_L$ due to the Assumption 1. Therefore this contradicts with the $W_t^L = V_L$ at $\pi'$.

(ii) Now, we need to show that when $\pi_t < \pi^*$, $W_t^L < V_L$. Suppose not, there exists $t$ such that $W_t^L(\pi_2) \geq V_L$. Now consider the first case where $W_t^L(\pi_2) > V_L$. I claim that in this case, there exists $\pi''$ such that $W_t^L(\pi'') = V_L$. If not, then for all $\pi_t < \pi_2$, we must have $W_t^L(\pi_t) > V_L$, then low quality asset trades at rate $\lambda_L = 0$, where high quality asset trades at rate$\lambda\mu$, hence the market belief goes down, bu t since the market belief is bounded below, then there exists $\pi'_\infty$ such that $\pi_t \to \pi'_\infty > 0$, but we have shown that the stationary belief is unique, hence the this is a contradiction. Hence, we can find a market belief $\pi''_t$ such that $W_t^L(\pi''_t) = V_L$, and consider the continuation dynamics at $\pi''_t + \varepsilon'$, since the market belief will go down due to the fact that the uninformed buyer will not trade with anybody. And

$$rW_t^L = rC_L + \frac{\mathrm{d}W_t^L}{\mathrm{d}t}$$

Since $W_t^L > V_L$ at $\pi''_t + \varepsilon$, then the continuation value goes up, therefore, the market belief goes down. Due to the fact that the market belief is bounded below, we can claim that $\pi_t \to \pi''_\infty$, but this is also a contradiction. Hence we cannot have $\pi''$ such that $W_t^L(\pi_t) > V_L$ for $\pi_t < \pi^*$

□

## Proof of Corollary 2

*Proof.* Following Lemma 6, we know that when $\pi_t > \pi^*$, $W_t^L > V_L$, therefore low-quality seller will not trade with informed buyer, since the uninformed buyers will bid $C_H$ for sure when $\pi_t > \pi^*$, then they will trade with both type of sellers in the market. Thus, high-quality seller is trading at rate $\lambda$, while low-quality seller is trading at rate $\lambda(1-\mu)$. Similarly, suppose that $\pi_t < \pi^*$, then there is no delay trading for low-quality seller, they will accept any price that is slightly higher than their reservation value, hence they are trading at rate $\lambda$. For high-quality seller, they only trade with informed buyers, since only informed buyers are willing to pay $C_H$ which is the reservation value for high-quality seller, hence we see that high-quality seller will only trade at rate $\lambda\mu$. □

## Proof of Proposition 2

(i) When $\pi_t < \pi^*$,$W_t^L < V_L$, the trading rate for low-quality $\lambda_t^L = \lambda$ and $\lambda_t^H = \lambda(1-m)$, then by continuation dynamics, we have

$$\mathrm{d}\pi_t = \lambda(1-m)\pi_t(1-\pi_t)\,\mathrm{d}t$$
$$rW_t^L = rC_L + \frac{dW_t^L}{\mathrm{d}t}$$

with boundary condition

$$\pi_{t_1^*} = \pi^*$$
$$W_{t_1^*}^L = V_L$$

Solving these equations give the result that

$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{-\lambda(1-m)t}} & t < t_1^* \\ \pi^* & t > t_1^* \end{cases}$$

$$W_t^L = \begin{cases} C_L + e^{-r(t_1^*-t)}(V_L - C_L) & t < t_1^* \\ V_L & t > t_1^* \end{cases}$$

(ii) When $\pi_t > \pi^*$, $W_t^L > V_L$, hence the trading rate for low-quality seller $\lambda_t^L = \lambda(1-m)$ and $\lambda_t^H = \lambda$, then by continuation dynamics, we have

$$\mathrm{d}\pi_t = -\lambda\mu\pi_t(1-\pi_t)\,\mathrm{d}t \tag{66}$$
$$rW_t^L = rC_L + \lambda(1-m)(C_H - W_t^L) + \frac{dW_t^L}{\mathrm{d}t} \tag{67}$$

With boundary condition

$$\pi_{t_2^*} = \pi^*$$
$$W^L_{t_2^*} = V_L$$

$$\pi_t = \begin{cases} \dfrac{\pi_0}{\pi_0 + (1-\pi_0)e^{-\lambda\mu t}} & t < t_2^* \\ \pi^* & t > T_2^* \end{cases} \tag{68}$$

$$W^L_t = \begin{cases} V_L + \left(1 - e^{-(r+\lambda)(T_2^*-t)}\right)(C_H - V_L) & t < t_2^* \\ V_L & t > T_2^* \end{cases} \tag{69}$$

## Proof of Proposition 3

1. First, we consider the case that $.\pi_0 > \pi^*$ For the high-quality seller, the trading rate is $\lambda m$ when $t < t_1^*$ and$\lambda m + \gamma^*$. Hence the average trading time for high-quality asset can be expressed as

$$\tau_H = \int_0^{t_1^*} s\lambda m e^{-\lambda m s} ds + e^{-\lambda\mu t_1^*}\frac{1}{\lambda m + \gamma^*}$$

And we can simplify the expression and get that

$$\tau_H = (1 - e^{-\lambda\mu t_1^*})\frac{1}{\lambda\mu} + e^{-\lambda\mu t_1^*}\frac{1}{\lambda m + \gamma^*}$$

Similarly, for the low-quality asset, the trading rate is $\lambda$ when$t < t_1^*$ and $\lambda\mu + \gamma^*$ when $t \geq t_1^*$. Therefore, the average trading time for low-quality asset can be expressed as

$$\tau_L = \int_0^{t_1^*} s\lambda e^{-\lambda s} ds + e^{-\lambda t_1^*}\frac{1}{\lambda m + \gamma^*}$$

which, again can be simplified as

$$\tau_L = (1 - e^{-\lambda t_1^*})\frac{1}{\lambda} + e^{-\lambda t_1^*}\frac{1}{\lambda m + \gamma^*}$$

2. When $\pi_0 > \pi^*$, following the same calculation, we can get the results.

## Proof of Corollary 3

I prove the two cases separately.

1. When $\pi_0 < \pi^*$, I can rewrite (32)(33) as

$$\tau_H = \frac{1}{\lambda\mu}(1 - e^{\lambda\mu T_1^*}) + e^{-\lambda\mu T_1^*}\frac{1}{\lambda\mu + \gamma^*}$$
$$\tau_L = \frac{1}{\lambda}(1 - e^{-\lambda T_1^*}) + e^{-\lambda T_1^*}\frac{1}{\lambda\mu + \gamma^*}$$

   Since $\lambda > \lambda\mu + \gamma^* > \lambda\mu$, it is easy to see that the weighted average of $\frac{1}{\lambda}$ and $\frac{1}{\lambda\mu+\gamma^*}$ should be lower than the weighted average of $\frac{1}{\lambda\mu}$ and $\frac{1}{\lambda\mu+\gamma^*}$, which is $\tau_L < \tau_H$.

2. When $\pi_0 > \pi^*$, we can applying the same logic, and using the fact that $\lambda > \gamma^* + \lambda\mu > \lambda(1-\mu)$, then it easy to show that in this case $\tau_L > \tau_H$

# Appendix B Equilibrium with homogeneous buyers

In this section, we characterize the equilibrium with homogeneous buyers, where all the buyers in the market are equally uninformed about the asset quality.

Given any offer $p$ from the buyer, if $\mu^H(p) \geq 0$, then $\mu^L(p) > 0$, therefore, in the equilibrium, $\lambda_t^L \geq \lambda_t^H$. That is, the market belief about the asset quality is non-decreasing conditional no trade occurs.

Similarly, we can write down the continuation value dynamics for low-type seller

$$rW_t^L = rC_L + \gamma_t(C_H - W_t^L) + \frac{\mathrm{d}W_t^L}{\mathrm{d}t} \tag{70}$$

Define $\pi^\dagger$ as the belief threshold such that

$$\pi_t(V_H - C_H) + (1-\pi_t)(V_L - C_H) = \max\{(1-\pi_t)(V_L - W_t^L), 0\} \tag{71}$$

where at $\pi^\dagger$, the buyer is indifference between offering $C_H$ and $W_t^L$.

$$W^L = \mathbb{E}\left\{(1-e^{-r(\tau-t)})C_L + e^{-r(\tau-t)}C_H\right\} \tag{72}$$

$$= \frac{rC_L + \lambda\gamma C_H}{r + \lambda\gamma} \tag{73}$$

where $\gamma$ is the probability that $C_H$ is offered in the equilibrium.

There are essentially multiple equilibrium in this model with homogeneous buyers, for simplicity, I only focus on the Markov strategy, where the strategies for both sellers and buyers are a function of market belief about the asset quality, and we allow mixed strategies for both sellers and buyers.

**Proposition 6.** *Given $\pi_0$, there exist a equilibrium such that*

- *when $\pi_t < \pi^*$, $\sigma_t(W_t^L) = 1$*
- *when $\pi_t = \pi^*$,*

$$\sigma_t(C_H) = \gamma^\dagger \tag{74}$$

$$\sigma_t(p) = 1 - \gamma^\dagger, \quad \forall p < V_L \tag{75}$$

*where $W_t^L$ is characterized by*

$$W_t^L = C_L + e^{-r(t^*-t)}(V_L - C_L) \tag{76}$$

*and $t^*$ is given by*

$$t^* = \frac{1}{\lambda}\left(\log(\frac{\pi^*}{1-\pi^*}) - \log(\frac{\pi_0}{1-\pi_0})\right) \tag{77}$$

- *when $\pi_t > \pi^0$, $\sigma_t(C_H) = 1$.*

So in this equilibrium when $\pi_t < \pi_0$, the uninformed buyer is offering $W_t^L$ to target the low-type seller, and the low-type seller accepts this offer with probability 1. When $\pi_t > \pi_0$, the buyers randomize between pooling offer $C_H$ and non-serious offer $p < V_L$ and both types of seller in this case only accept the pooling offer $C_H$, and reject non-serious offer for sure. The reason for mixing between pooling offer and non-serious offer at $\pi^*$ is to make sure that the low-type seller is indifferent between accepting $W_t^L$ before $\pi_t$ reaches $\pi^*$, otherwise, she always has incentive to reject $W_t^L$ before $t^*$ and wait for the pooling offer $C_H$ for sure. And the belief will stop at $\pi^*$, since given the strategy for the buyers, the trading rates for two types of asset are the same, therefore, belief is constant conditional on no trade occurs.

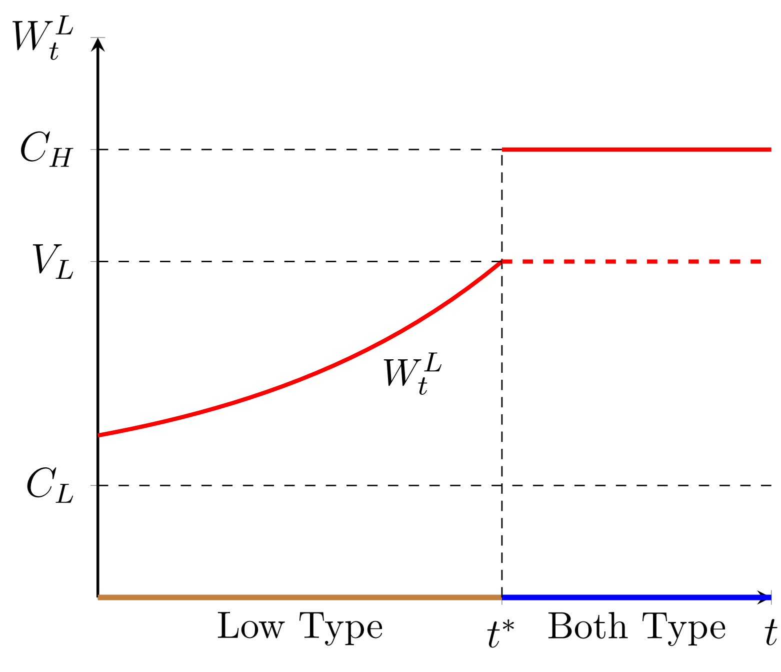


Figure 7: Equilibrium with Homogeneous buyers